Far-field contribution of evanescent modes to the electromagnetic Green tensor

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Understanding the behavior of the evanescent part of the electromagnetic field has important implications in many branches of modern physics, such as near-field optics. Motivated by recent disagreement in the literature, we derive an expression for the far-field asymptotic behavior of the free-space electromagnetic Green tensor. In this communication we explicitly obtain the exact result:

\[ G_0^\infty(x, \omega) = \frac{1}{2\pi} \int d^2k \frac{1}{\beta(k_1, \omega)} \left[ \frac{\omega}{r \cos \theta} \right] \frac{1}{c} \frac{\omega r \sin \theta}{r \cos \theta} \left[ 1 - \frac{\sin \theta}{(\omega/c)^2 \cos^2 \theta} \right] e^{ik \cdot x} + O(r^{-7/2}), \quad (\omega/c)r \gg 1. \]

To verify the correctness of asymptotic expression (6), we evaluated it and the integral in Eq. (4) numerically and found them to be in excellent agreement for large values of \((\omega/c)r\).
On the $x_3$ axis ($\theta = 0, \text{ or } \pi$) Eq. (6) gives the exact result, viz.,

$$G_0^{\eta}(\mathbf{x}, \omega)|_{\theta=0,\pi} = \frac{1}{r}, \quad (7)$$

as is well known in the scalar theory. In contrast to the conclusions reached in Refs. 1–4, it is also clear from Eq. (6) that the $1/r$ behavior found on the $x_3$ axis does not hold for arbitrary directions. The directions specified by $\theta = 0$ and $\theta = \pi$ are special, as elaborated in Refs. 6 and 11. The off-axis behavior of $G_0^{\eta}(\mathbf{x}, \omega)$ is $r^{-\frac{3}{2}}$. This clearly indicates that the evanescent waves do not contribute to the radiated energy in the far zone.

Finally, using Eq. (1), we give the leading term in the far-field contribution from the evanescent modes to the Green tensor:

$$G_{\alpha\beta}^{\eta}(\mathbf{x}, \omega) = \frac{1}{r} \frac{1}{\cos \theta} \left[ \frac{\omega}{c} r \sin \theta \right] + O(r^{-5/2}),$$

as in the scalar case, $G_{\alpha\beta}^{\eta}(\mathbf{x}, \omega)$ decays off axis in the far zone as $r^{-3/2}$.

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In conclusion, the authors hope that this communication resolves the controversy in the literature regarding the far-field asymptotic behavior of the evanescent part of the electromagnetic Green tensor.

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**REFERENCES**