Coherence of Leaky-Mode Vertical-Cavity Surface-Emitting Laser Arrays

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***Abstract*— The coherence of leaky-mode vertical-cavity surface-emitting laser arrays is analyzed using a stochastic coupled-mode theory. The conditions under which the theory is applicable to leaky-mode arrays is established. Measurements are performed on fabricated arrays, which serve to verify the predictions of the stochastic theory. Theory and experiment demonstrate that the critical determinants of array coherence are uniformity among the elements and control of the array supermodes.**

***Index Terms*— Optical coupling, semiconductor laser arrays, surface-emitting lasers.**

I. INTRODUCTION

VERTICAL-CAVITY surface-emitting lasers (VCSELs) can be organized into two-dimensional laser arrays. In such a configuration, the lasers can be coupled to form a coherent collection of emitters. These coherently coupled VCSEL arrays are of interest for a variety of applications in high-power pumping, sensing, and targeting, since they can be designed to operate in a single mode with significantly higher power than an individual emitter. Other appealing coherent VCSEL array properties include electronic beam steering [1], [2] and low beam divergence [3], [4].

To describe the behavior of coherent laser arrays many approaches have been pursued [5], [6], especially coupled mode theory [7], [8], which is appropriate for evanescent coupling. Unfortunately, coupled mode theory is not formulated such that it can predict partially coherent array behavior. Attempts have been made to address this issue, including a stochastic harmonic oscillator model [9], but the properties of the spatial modes cannot be predicted *ab initio*. Recently, we developed a more complete stochastic coupled waveguide model that directly predicts the modal behavior of VCSEL arrays [10]. However, evanescent coupling assumed in coupled mode theory is not relevant for leaky mode arrays [11], [12].

Using our stochastic coupled mode theory, we analyze the coherence of implant-defined VCSEL arrays. We first develop a theoretical analog to describe approximately the leaky-mode behavior of the coherent VCSEL arrays. This model is based on the observed near-field intensity of the array modes. Using this with the stochastic theory, we are able to predict the array coherence. A far-field intensity profiler and an optical tabletop imaging spectrometer are used to characterize the properties of 1×2 implant-defined VCSEL arrays. These measurements are in agreement with the theory and reveal properties of the optical coupling. This analysis is valuable for the design of coherently coupled laser arrays.

II. THEORY

In our previous work [10], we used a coupled mode theory of evanescently coupled Fabry-Perot resonator arrays with random initial conditions. From this, we derived the coherence matrix for a two-element array seeded by two mutually incoherent fields. The degree of coherence, γ, of the coupled lasers is expressed in terms of the intensities of the two supermodes of the two waveguides:

$$|\gamma| = \left| \frac{\left[I_a^+ I_b^+\right]^{1/2} - \left[I_a^- I_b^-\right]^{1/2}}{\left[\left(I_a^+ + I_a^-\right)\left(I_b^+ + I_b^-\right)\right]^{1/2}} \right|, \tag{1}$$

where a and b refer to the individual laser elements and $+$ and $-$ refer to the in-phase and out-of-phase supermodes, respectively. Using these expressions, we can predict the degree of coherence as a function of the relative mode intensities and the distribution of intensity between the two elements (determined by the array asymmetry and the coupling strength). We showed that for any practical array where asymmetry is unavoidable and both supermodes can operate, strong coupling is necessary to maintain high coherence.

Considering the eigenmode solutions of the coupled mode theory allows for further modification of Eqn. 1 and a better understanding of the applicability of this result. In our work and in general, the vector representations of the eigenmode solutions are [10], [13]

$$v_+ = N \begin{bmatrix} K_{ab} \\ \Delta + \psi \end{bmatrix} = N' \begin{bmatrix} R \\ 1 \end{bmatrix}, \tag{2}$$

$$v_- = N \begin{bmatrix} \Delta + \psi \\ -K_{ba} \end{bmatrix} = N' \begin{bmatrix} 1 \\ -R^* \end{bmatrix}, \tag{3}$$

where N and N' are normalization factors, $K_{ab} = K_{ba}^*$ are the coupling constants, Δ is the difference in effective indices or propagation constants between the two guides, $\psi = \left[\Delta^2 + K_{ab}K_{ba}\right]^{1/2}$, and we define

$$R = \frac{K_{ab}}{\Delta + \psi}. \tag{4}$$

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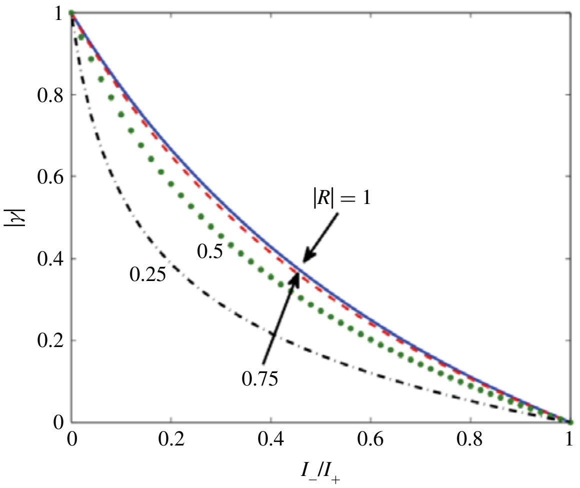
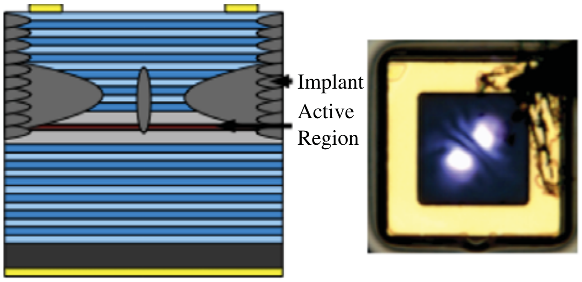


Fig. 1. Degree of coherence as a function of mode intensities for different array mode asymmetries.



(a) (b)

Fig. 2. (a) Cross-sectional schematic and (b) near-zone image of the implant-defined VCSEL array tested.

$|R|$ is then a measure of the mode asymmetry (i.e. if $|R| = 1$ then the intensity profiles of the in-phase and out-of-phase modes are perfectly symmetric and anti-symmetric, respectively).

The measured mode intensities yield the weighting on each mode of Eqns. 2 and 3. In other words, if the measured total intensities of the two modes are I_+ and I_-, then the corresponding eigenmodes are $\sqrt{I_+}v_+$ and $\sqrt{I_-}v_-$. Using the definitions in our previous paper, the intensities in the individual waveguides for the two modes are the measured intensity times the square of the corresponding vector element or

$$\begin{aligned} I_a^+ &= I_+ N'^2 \left|R\right|^2, \\ I_b^+ &= I_+ N'^2, \\ I_a^- &= I_- N'^2, \\ I_b^- &= I_- N'^2 \left|R\right|^2. \end{aligned} \tag{5}$$

It can be seen that $I_a^+/I_b^+ = |R|^2$ and $I_a^-/I_b^- = 1/\left|R\right|^2$, and Eqn. 1 then can be rewritten as

$$|\gamma| = \left| \frac{|R|\left(1 - I_-/I_+\right)}{\left[\left(|R|^2 + I_-/I_+\right)\left(1 + I_-/I_+\left|R\right|^2\right)\right]^{1/2}} \right|. \tag{6}$$

When the coupling strength is great enough or the array asymmetry is small enough (i.e. $K_{ab} \gg \Delta$ and $|R| \to 1$), there is a fundamental limit to the coherence given by

$$|\gamma| = \left| \frac{I_+ - I_-}{I_+ + I_-} \right|. \tag{7}$$

Eqn. 6 clearly illustrates the change in the degree of coherence as the mode asymmetry ($|R|$) and mode intensities (I_+ and I_-) change. The form of Eqn. 6 makes it easier to see the limits of high coherence. The degree of coherence is plotted for different values of the mode ratios in Fig. 1. Even for $|R| = 0.75$, the degree of coherence deviates little from the maximum limit. Thus, it is evident that there is a wide range of mode asymmetries for which the array fields have high coherence.

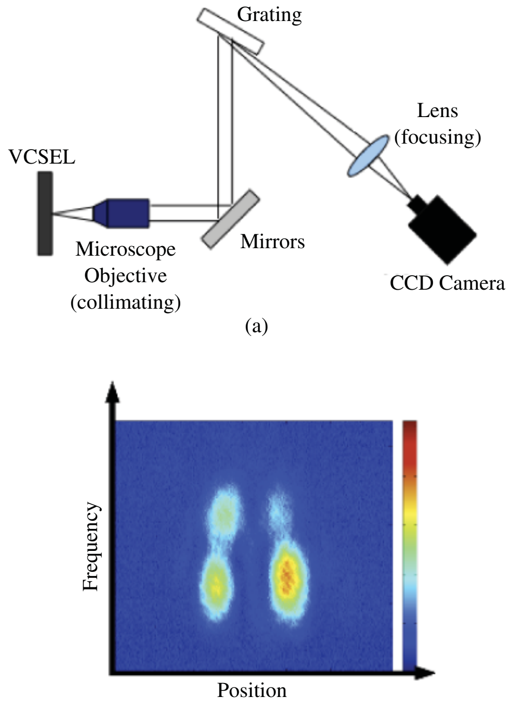
The conventional coupled mode theory that is the basis of this analysis is generally applicable only to evanescently coupled fields [11]. The implant-defined coherent VCSEL arrays that we seek to analyze, however, have recently been demonstrated to be leaky-mode coupled, i.e. light is shared between elements with propagating fields rather than evanescently decaying fields [12]. However, since the stochastic theory is based on the modes, the representation of the coupled modes is more important than the coupling mechanism. In short, if the behavior of the modes is reasonably well approximated by coupled mode theory, then the stochastic theory should be applicable.

In order to apply the stochastic coupled mode theory to leaky-mode arrays, the array modes must consist of two modes that approximately follow the form in Eqns. 2 and 3. In this work, arrays with the lowest order out-of-phase (no central fringe) and in-phase (one central fringe) modes operating are measured. The field fringe in the coupling region between the laser elements is apparent in the near-field image in Fig. 2b. If it is possible to neglect the central fringe of the in-phase mode (e.g. if the central lobe intensity is negligible in comparison to the two outer waveguide intensities) then it can be possible for the modes to be approximated by two-element vectors that behave as those in coupled mode theory. It will be shown in the next section that the arrays studied satisfy these criteria, and thus we can directly apply our stochastic coupled mode analysis.

III. EXPERIMENT

Experiments are performed on 1×2 implant-defined VCSEL arrays similar to those described in previous work [12], [14]. Fig. 2a shows a cross-section illustration of the array, which is defined using ion implantation damage to electrically isolate the different elements. This separates the array into different current paths that simultaneously pixelate the gain and create a thermal- and carrier-induced index difference [12], [15]. A near-zone image of an array showing two separate elements is shown in Fig. 2b. Multiple measurements of the near- and far-field profiles are taken at different injection currents, since the modes change with current. In Fig. 2b there is only one mode, but a second array mode begins to turn on as the current is increased.

Coherence experiments are performed by directly measuring the fringe visibility [16], [17] and by measuring the mode intensities. Using a grating spectrometer (setup shown in Fig. 3a), we collect spectrally resolved images of the near-field modes of a 2×1 implant-defined VCSEL array (Fig. 3b).



(a)

(b)

Fig. 3. (a) Tabletop imaging spectrometer setup and (b) spectrometer data showing the two array supermodes.

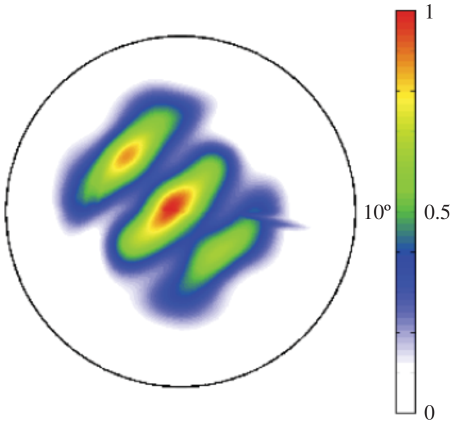


Fig. 4. Far-field radiation profile taken from the implant-defined VCSEL array tested.

These measurements provide the mode intensities needed in Eqn. 1. Using a goniometric radiometer, we are also able to image the far-field radiation pattern from which the coherence can be directly measured [17]. A far-field profile from the goniometric radiometer that corresponds to the spectrometer data in Fig. 3b is shown in Fig. 4.

As already discussed, the leaky modes must satisfy the conditions in Eqns. 2 and 3 in order to be treated using the stochastic coupled mode theory. In particular, it must be found that $\left[I_a^+/I_b^+\right]^{1/2} = \left[I_b^-/I_a^-\right]^{1/2} = |R|$ for the stochastic coupled mode theory to be applied. A plot of the measured $\left[I_a^+/I_b^+\right]^{1/2}$ and $\left[I_b^-/I_a^-\right]^{1/2}$ as a function of the ratio of the total mode intensities (which changes with injection current) is given in Fig. 5. Ideally, the points at each current in the plot would overlap, and this would give an unambiguous value of $|R|$.

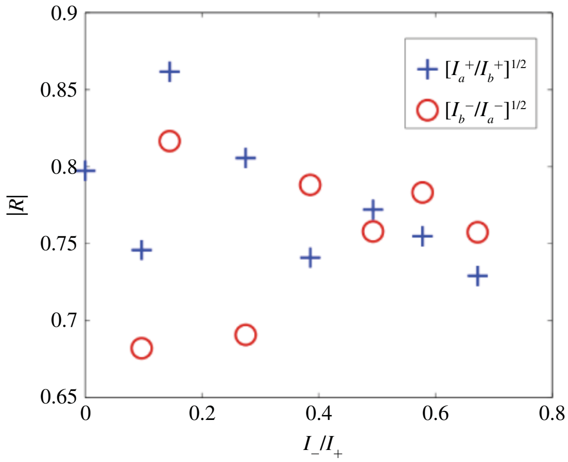


Fig. 5. Comparison of the measured mode asymmetry for the in-phase and out-of-phase modes.

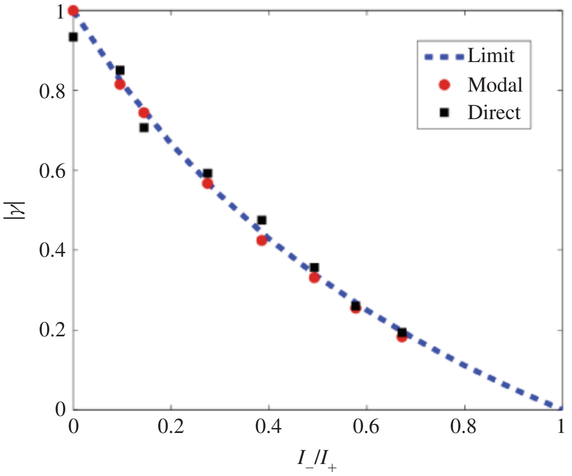


Fig. 6. Comparison of the calculated and directly measured degree of coherence with the theoretical limit indicated.

In general, this condition is approximately satisfied for all data points. Discrepancies can be attributed to experimental error arising as a result of limitations of the spectrometer (sensitivities and resolutions of the lenses, grating, and charge-coupled device (CCD) camera). These data in Fig. 5 suggest that the stochastic coupled mode theory can be suitably applied to the implant-defined VCSEL arrays. Moreover, the value of $|R|$ tends to be above 0.7 for all mode ratios, which is in the range of the upper limit of coherence as illustrated in Fig. 1.

Using near-field spectrally resolved measurements with Eqn. 1 as well as direct measurements of the far-field pattern [17], the coherence is extracted. Fig. 6 shows the degree of coherence measured using these two approaches as a function of the ratio of the mode intensities. Excellent agreement is found between the two sets of data, which serves to verify the validity of the approximations as well as that of the stochastic coupled mode theory [10]. In particular, this result reveals that the predicted reduction in coherence of a coupled laser array is a direct result of the transition from single-mode to multimode operation.

The theoretical high-coherence limit, given by Eqn. 7, is also indicated by the dotted line in Fig. 6. The measured coherence is observed to lie along this limit, which suggests that the implanted VCSEL arrays operate in a high-coherence regime, consistent with the results of Fig. 5. High coherence is in fact expected for leaky-mode laser arrays [11], and our analysis presented here accurately characterizes the coupling properties of the implanted coherent VCSEL arrays.

IV. CONCLUSION

We have used a stochastic coupled mode theory to analyze implant-defined coherent VCSEL arrays. Despite the fact that the arrays support leaky modes, we show that the coupled mode theory provides an adequate approximation to the coherence behavior of the array. By using a tabletop imaging spectrometer and a far-field profiler, we perform measurements on the implant arrays. These measurements serve to verify the predictions of the stochastic theory that coherence is determined by the number and intensities of the array modes. In addition, the analysis reveals that the arrays are operating in a strong coupling regime, as is to be expected for leaky-mode arrays.

This work has shown that the stochastic coupled mode theory is generally applicable to a variety of systems. Not only should the theory be useful for the evanescently coupled arrays for which it was designed, but here we have demonstrated it to be useful for leaky-mode arrays. This suggests that the theory could be useful for a wider class of laser arrays than originally intended. Our theory supports the result that determinants of array coherence are uniformity among the elements and control of the array supermodes.

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