Power-excitation diffraction tomography with partially coherent light

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Some consequences of using partially coherent fields in the recently proposed method of power-extinction diffraction tomography are analyzed. It is found that the method is very tolerant of short spectral coherence lengths. The spectral coherence length of the field is shown to set the scale of a low-pass filter that acts on the subject. The implications of these results for implementation of the method are discussed.

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We investigate the effects of using partially coherent rather than fully coherent radiation in a new method of diffraction tomography, based on the use of a generalization of the optical cross-section theorem.[1] We make use of measurements of the power extinguished on scattering from weakly scattering objects to obtain three-dimensional reconstructions of the object structure. This technique may be called power-extinction diffraction tomography. One of the principal advantages of this technique is that unlike most other inverse scattering methods (see, for example, Refs. 2–5) it does not require that the phase of the scattered field be measured. Instead, power-extinction diffraction tomography relies on interference within the domain of the scatterer. As in holographic techniques,[5] it is critical to understand the influence of the coherence of the probe field on the reconstructed object.

It was shown in Ref. 1 that a data function, $D(\mathbf{s}_1, \mathbf{s}_2)$, can be determined from measurements of the extinguished power and is related to the scattering amplitude, $f(\mathbf{s}_1, \mathbf{s}_2)$, of the scattering object of the formula

$$D(\mathbf{s}_1,\mathbf{s}_2) = f(\mathbf{s}_1,\mathbf{s}_2) - f^*(\mathbf{s}_2,\mathbf{s}_1)\,, \tag{1}$$

where $\mathbf{s}_1$ and $\mathbf{s}_2$ denote unit vectors in the direction of propagation of the incident beams (see Fig. 1). Let $P^{(e)}(l)$ denote the power extinguished from two coherent plane waves of amplitude A_0; l denotes the path-length difference between the two beams. The data function $D(\mathbf{s}_1,\mathbf{s}_2)$ may be determined from four measurements of the extinguished power by use of the formula

$$D(\mathbf{s}_1,\mathbf{s}_2) \equiv \frac{k}{8\pi A_0^2}\left\{P^{(e)}\!\left(\frac{\pi}{2k}\right) - P^{(e)}\!\left(\frac{-\pi}{2k}\right) + i\left[P^{(e)}(0) - P^{(e)}\!\left(\frac{\pi}{k}\right)\right]\right\}. \tag{2}$$

For weakly scattering objects, the solution to the inverse problem, i.e., to the problem of determining the structure of the scattering object from measurements of the scattered field, is reduced to taking a Fourier transform of the data function. If $\alpha(\mathbf{r})$ denotes the imaginary (absorptive) part of the dielectric susceptibility of the object, then, within the accuracy of the first-order Born approximation, the data function is related to the Fourier transform of α by the formula

$$D(\mathbf{s}_1,\mathbf{s}_2) = 2ik^2 \int \mathrm{d}^3 r\,\alpha(\mathbf{r})\exp[-ik\mathbf{r}\cdot(\mathbf{s}_1 - \mathbf{s}_2)], \tag{3}$$

valid for all real unit vectors $\mathbf{s}_1$ and $\mathbf{s}_2$. Clearly, a low-pass version of the absorptive part $\alpha(\mathbf{r})$ may be reconstructed by inversion of formula (3).

A simple scheme for implementing this technique is illustrated in Fig. 1. A beam is incident on a beam splitter (BS), and two identical beams are produced, propagating in different directions. One beam is redirected by means of a mirror (M) so that the two beams are incident on the scatterer in different directions. The relative phase and propagation angles of the two beams may be controlled by the position and

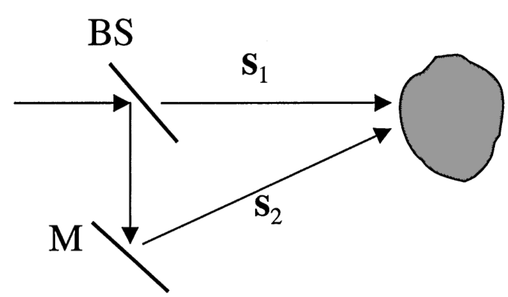

Fig. 1. Scheme for generating two mutually coherent beams needed to determine the data function. Two identical beams are generated at the beam splitter (BS). The final direction of propagation of the second beam and path-length difference between the beams is controlled by the mirror (M). The beams are finally incident on the object with directions given by $\mathbf{s}_1$ and $\mathbf{s}_2$.

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orientation of mirror M. The two beams differ from each other only in that one is a rotated version of the other and may have propagated some additional distance. Thus, in any particular realization, one beam can be represented as a rotated and translated version of the other.

We will investigate the effects of partial coherence by means of a numerical simulation of the scattering experiment and reconstruction of a spherical scatterer. Because of the spherical symmetry, the scattering amplitude $f(\mathbf{s}_1, \mathbf{s}_2)$ depends only on the angle between the two incident beams, and consequently we may write $f(\mathbf{s}_1, \mathbf{s}_2) \equiv F(\mathbf{s}_1 \cdot \mathbf{s}_2)$. The beams will be assumed to be of a Gaussian–Schell model form (Sec. 5.6 of Ref. 6). Each may be described by an angular correlation function, $\mathcal{A}_0(\mathbf{s}'_\perp, \mathbf{s}''_\perp)$ (Sec. 5.6 of Ref. 6). The two-dimensional vectors, $\mathbf{s}'_\perp$ and $\mathbf{s}''_\perp$, are the vector projections of unit vectors $\mathbf{s}'$ and $\mathbf{s}''$, respectively; the plane of projection is taken perpendicularly to the axis of the beam. Let us denote the width of the beam at half-maximum of the intensity by σ_s, the spectral correlation length by σ_g, and the maximum intensity by A_0^2. The angular correlation function of one of the beams, with its axis along the z direction, is then given by the expression

$$\mathcal{A}_0(\mathbf{s}'_\perp, \mathbf{s}''_\perp) = \left(\frac{k^2 A_0}{2\pi}\right)^2 \Delta^2 \sigma_s^2 \exp\left\{\frac{1}{8}[-4k^2\sigma_s^2(\mathbf{s}'_\perp + \mathbf{s}''_\perp)^2 - k^2\Delta^2(\mathbf{s}'_\perp - \mathbf{s}''_\perp)^2]\right\}, \tag{4}$$

where

$$\frac{1}{\Delta^2} = \frac{1}{4\sigma_s^2} + \frac{1}{\sigma_g^2} \cdot \tag{5}$$

The other beam is associated with a direction of propagation $\mathbf{s}_2 = S\mathbf{s}_1 = S\hat{z}$, where S is a 3×3 rotation matrix. To separate the effects of a finite spectral coherence length from the effects of finite beam width, we will consider the limit as $\sigma_s \to \infty$. The power extinguished from the beams on scattering from the object is given by the formula

$$P^{(e)}(l) = P_1^{(e)} + P_2^{(e)} + P_{12}^{(e)}(l), \tag{6}$$

where $P_1^{(e)}$ is the power extinguished from the original beam alone, $P_2^{(e)}$ is the power extinguished from the rotated and translated version of the original beam, and $P_{12}^{(e)}$ represents the contribution arising from interference between the beams. In the limit when the beam is infinitely broad, $P_2^{(e)} = P_1^{(e)}$.

The cross terms can be expressed in the form

$$P_{12}^{(e)}(l) = \frac{4\pi}{k}\,\mathrm{Im}\left\{\iint \mathcal{A}_0(\mathbf{s}'_\perp, \mathbf{s}''_\perp)\exp[ikl\mathbf{s}_1 \cdot (S\mathbf{s}'')] \times f(\mathbf{s}', S\mathbf{s}'')\mathrm{d}^2 s'_\perp \mathrm{d}^2 s''_\perp + \iint \mathcal{A}_0^*(\mathbf{s}'_\perp, \mathbf{s}''_\perp) \times \exp[-ikl\mathbf{s}_1 \cdot (S\mathbf{s}'')]f(S\mathbf{s}'', \mathbf{s}')\mathrm{d}^2 s'_\perp \mathrm{d}^2 s''_\perp\right\}, \tag{7}$$

where Im denotes the imaginary part. Under our assumptions that the beams are of the Gaussian–Schell model type and that the scatterer is spherically symmetric, Eq. (7) reduces to

$$P_{12}^{(e)}(l) = \frac{8\pi}{k}\,\mathrm{Im}\iint \mathcal{A}_0(\mathbf{s}'_\perp, \mathbf{s}''_\perp)\cos(kt\hat{z} \cdot \mathbf{s}'') \times F[\mathbf{s}' \cdot (S\mathbf{s}'')]\mathrm{d}^2 s'_\perp \mathrm{d}^2 s''_\perp . \tag{8}$$

We next expand the scattering amplitude $F(\mathbf{s}_1 \cdot \mathbf{s}_2)$ in a power series about $\mathbf{s}'_\perp = \mathbf{s}''_\perp = 0$ and find that in the neighborhood of this point

$$F[\mathbf{s}' \cdot (S\mathbf{s}'')] \approx F(\cos\theta) + \left[\frac{(s'_y + s''_y - s'_x - s''_x)}{\sqrt{2}}\sin\theta - \frac{(s'_y - s''_y - s'_x + s''_x)^2}{4}\cos\theta - \frac{(s'_x - s''_x + s'_y - s''_y)^2}{4}\right]F'(\cos\theta), \tag{9}$$

where θ is the angle of rotation associated with S, i.e., the angle between the axes of propagation of the beams. If the magnitude of the translation vector is small compared with Δ so that $l \ll \Delta$, the integrand of Eq. (8) is significant only for small values of $|\mathbf{s}'_\perp|$ and $|\mathbf{s}''_\perp|$, and consequently the z component of the unit vector can be replaced with the approximate expression (equivalent to the paraxial approximation)

$$\sqrt{1 - \mathbf{s}_{j\perp}^2} \approx 1 - \frac{1}{2}\mathbf{s}_{j\perp}^2. \tag{10}$$

The cross term is then given by the expression

$$P_{12}^{(e)}(l) = \frac{8\pi}{k} A_0^2\left\{\mathrm{Re}\left(\frac{e^{iL}}{1 + iL/\delta^2}\right)\mathrm{Im}\,F(\cos\theta) + \frac{1 + \cos\theta}{2}\,\mathrm{Re}\left[\frac{e^{iL}}{\delta^2(1 + iL/\delta^2)^2}\right]\mathrm{Im}\,F'(\cos\theta)\right\}, \tag{11}$$

where $L = kl$, $\delta = k\Delta$, and Re denotes the real part. The data function (2) is then given by the expression

$$D(\mathbf{s}_1, \mathbf{s}_2) = i\left\{(1 + \xi)\mathrm{Im}\,F(\mathbf{s}_1 \cdot \mathbf{s}_2) + \frac{1 + \mathbf{s}_1 \cdot \mathbf{s}_2}{2\delta^2} \times [1 + (1 - \pi^2/\delta^4)\xi^2]\mathrm{Im}\,F'(\mathbf{s}_1 \cdot \mathbf{s}_2)\right\}, \tag{12}$$

where $\xi \equiv (1 + \pi^2/\delta^4)^{-1}$. Hence for partially coherent (PC) beams, a low-pass filtered version of the absorptive part of the susceptibility, α_{PC}, can be reconstructed by use of a Fourier inverse transform. Explicitly,

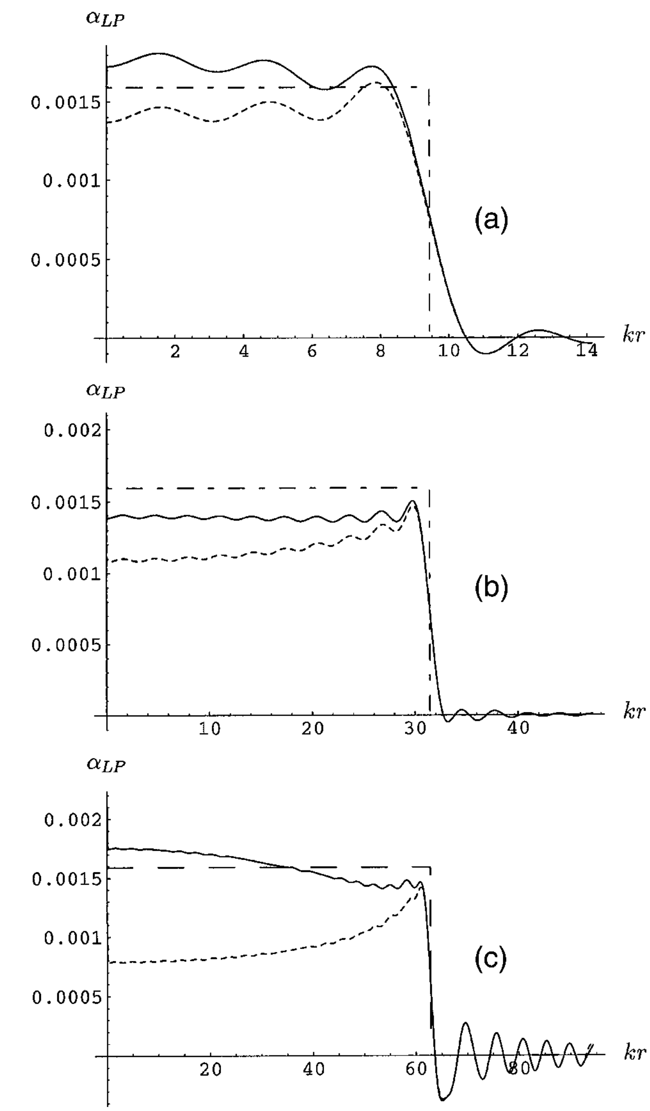

Fig. 2. Demonstrating the effects of partial coherence on the reconstruction of an absorbing sphere. In all cases the susceptibility is $\eta = 0.01i/2\pi$. In (a) $ka = 3\pi$, $k\Delta = 10\pi$ (dashed curve), and $k\Delta = 3\pi$ (solid curve). In (b) $ka = 10\pi$, $k\Delta = 10\pi$ (solid curve), and $k\Delta = 100\pi$ (dashed curve). In (c) $ka = 20\pi$, $k\Delta = 10\pi$ (solid curve), and $k\Delta = 100\pi$ (dashed curve).

$$\alpha_{\mathrm{PC}}(\mathbf{r}) = \frac{1+\xi}{2}\,\alpha_{\mathrm{LP}}(\mathbf{r}) + \frac{k[1 + (1 - \pi^2/\delta^4)\xi^2]}{64\delta^2\pi^3} \times \int_{|\mathbf{s}|\leq 2} (2 - s^2/2)\exp(ik\mathbf{r}\cdot\mathbf{s})\mathrm{Im}\,F'(1 - s^2/2)\mathrm{d}^3 s\,, \tag{13}$$

where α_{LP} denotes the low-pass-filtered reconstruction of α, discussed in Ref. 1. It may be shown that $\alpha_{\mathrm{PC}} = \alpha_{\mathrm{LP}}$ when the beams are fully spatially coherent, i.e., when $\sigma_g \rightarrow \infty$.

Computational results based on this theory are displayed in Fig. 2 and demonstrate the effects of partial coherence of the incident beams on the reconstruction of homogeneous spheres of various sizes and susceptibilities. Only the radial dependence of the imaginary part of the reconstructed susceptibility of the spheres is shown. It can be seen from the figures that the reconstruction is satisfactory even when the spectral coherence length of the field is comparable to the linear dimensions of the scatterer or even smaller. This is so because the overlapping beams remain mutually highly coherent and, when the angle between the beams is sufficiently small, the beams are only shifted from each other by a distance of the order of the wavelength. For larger angles between the beams, the cross terms in the extinguished power fall off more rapidly, so the finite spectral coherence length of the field acts as a low-pass filter.

We have examined a main question regarding the feasibility of power-extinction diffraction tomography. The advantage of this method over other reconstruction techniques is that no direct measurement of the phase of the scattered field is required. We have demonstrated that this technique could be used even when the incident beams have relatively short spectral coherence lengths. We have also shown that the minimum required coherence length is determined not by the size of the scatterer but rather by the central wavelength of the incident beam. This result implies that it may be possible to perform experiments of this type with relatively low-quality lasers.

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