JOURNAL OF MODERN OPTICS, 1999, VOL. 46, NO. 5, 891–899

The optical cross-section theorem with incident fields containing evanescent components

P. SCOTT CARNEY

Department of Physics and Astronomy and the Rochester Theory Center for Optical Science and Engineering University of Rochester, Rochester, New York 14627, USA

(*Received 8 April 1998; revised 4 October 1998*)

Abstract. The optical cross-section theorem is extended to cases in which the incident field contains evanescent waves. Physical interpretations are discussed. Some explicit examples are given and possible applications are proposed.

1. Introduction

The optical theorem is generally formulated for the case where the incident field is a single homogeneous plane wave [1, 2] and for pairs of plane waves incident on non-absorbing media [3, p. 47]. More recently the theorem has been generalized for arbitrary incident-free fields [4], that is fields containing homogeneous plane wave modes only. In this paper a generalization of the theorem is derived which applies when evanescent waves are present in the incident field.

The generalization of the optical theorem obtained in [4] has the form

$$P^{(e)} = \frac{4\pi}{k}\Im \int\int \mathcal{A}(\mathbf{s}'_{\perp}, \mathbf{s}''_{\perp}) f(\mathbf{s}', \mathbf{s}'')\, \mathrm{d}^2 s'\, \mathrm{d}^2 s'', \tag{1}$$

where

$$\mathcal{A}(\mathbf{s}'_{\perp}, \mathbf{s}''_{\perp}) = \langle a^*(\mathbf{s}'_{\perp}) a(\mathbf{s}''_{\perp})\rangle \tag{2}$$

and $P^{(e)}$ is the power extinguished from the incident beam by scattering and absorption. $\mathcal{A}$ is the angular correlation function of the incident field which is described statistically by an ensemble, each member of which is a superposition of plane waves with randomly distributed amplitudes $a(\mathbf{s}_{\perp})$ and $\Im$ denotes the imaginary part. Further, $f(\mathbf{s}', \mathbf{s}'')$ is the scattering amplitude in the direction specified by the unit vector $\mathbf{s}'$ when the incident wave is a plane wave which propagates in the direction specified by $\mathbf{s}''$. More explicitly, the total field is given by

$$\psi(\mathbf{r}) = \psi^{(i)}(\mathbf{r}) + \psi^{(s)}(\mathbf{r}), \tag{3}$$

where

Journal of Modern Optics ISSN 0950–0340 print/ISSN 1362–3044 online
http://www.tandf.co.uk/JNLS/mop.htm
http://www.taylorandfrancis.com/JNLS/mop.htm

$$\psi^{(i)}(\mathbf{r}) = \int a(s'_\perp) \exp(ik\mathbf{r}\cdot\mathbf{s}')\, d^2 s', \tag{4}$$

$$\psi^{(s)}(\mathbf{r}) = \int a(s'_\perp) f(\mathbf{s}, \mathbf{s}') \frac{\exp(ikr)}{r}\, d^2 s', \tag{5}$$

$$\mathbf{s} = \frac{\mathbf{r}}{r} (r \equiv |\mathbf{r}|), \tag{6}$$

$$\mathbf{s}' = p\hat{\mathbf{x}} + q\hat{\mathbf{y}} + (1 - p^2 - q^2)^{1/2}\hat{\mathbf{z}}, \tag{7}$$

$$\mathbf{s}'_\perp = p\hat{\mathbf{x}} + q\hat{\mathbf{y}}, \tag{8}$$

$$d^2 s' = dp\, dq, \tag{9}$$

and $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are the Cartesian unit vectors in the x, y and z directions respectively. The absorbed power is given by the expression

$$P^{(a)} = -\frac{1}{k} \Im \int_\Sigma (\psi^* \nabla\psi) \cdot \mathbf{s}\, d\Sigma, \tag{10}$$

where Σ is a surface which completely enclosed the scatterer. The power carried by the scattered field is given by the expression

$$P^{(s)} = \frac{1}{k} \Im \int_\Sigma (\psi^{(s)*} \nabla\psi^{(s)}) \cdot \mathbf{s}\, d\Sigma. \tag{11}$$

Noting that since $\psi^{(i)}$ satisfies the homogeneous Helmholtz equation

$$\Im \int_\Sigma (\psi^{(i)*} \nabla\psi^{(i)}) \cdot \mathbf{s}\, d\Sigma = 0, \tag{12}$$

we find that for any deterministic field the extinguished power is given by the expression

$$P^{(e)} = -\frac{1}{k} \Im \int_\Sigma (\psi^{(i)*} \nabla\psi^{(s)} + \psi^{(s)*} \nabla\psi^{(i)}) \cdot \mathbf{s}\, d\Sigma. \tag{13}$$

The theorem given by equation (1) has been derived by integrating the right-hand side of equation (13) over the surface Σ of a large sphere of radius R, centred at some point in the scatterer. The extinguished power is independent of the size of the sphere over which the flux is integrated so long as the sphere encloses the scatterer. For kR sufficiently large the integral may be evaluted by the method of stationary phase [5]. Such a derivation is valid only with free incident fields, that is with incident fields of the form given by equation (4) for which the angular spectrum amplitudes $a(\mathbf{s}_\perp)$ may be non-zero only for real unit vectors $\mathbf{s}$ so that

$$\mathbf{s}_\perp^2 \equiv p^2 + q^2 \leqslant 1. \tag{14}$$

It should be noted that, although equation (1) is a generalization of what is usually referred to as the optical cross-section theorem, we have abandoned the concept of cross-section and instead deal with the unnormalized extinguished power $P^{(e)}$. The cross-section is defined as the extinguished power divided by the incident power per unit area. The incident power per unit area for anything other than a single plane wave is an ambiguous quantity which depends not only on the orientation of the plane of projection but also on its absolute position.

2. Derivation of the theorem for evanescent incident fields

We shall generalize the theorem expressed by equation (1) to include any incident field which propagates into the half-space $z > 0$. To do this we shall first extend the optical theorem to situations where the incident field is evanescent, that is fields for which the incident plane wave has the form

$$\psi^{(i)} = \exp(ik\mathbf{r}\cdot\mathbf{s}_0), \tag{15}$$

where the z component of the unit vector $\mathbf{s}_0$ ($\mathbf{s}_0 \cdot \mathbf{s}_0 = 1$) is strictly imaginary. Such a wave decays exponentially in amplitude with increasing values of z.

The extinguished power in this case is still taken to be the sum of the absorbed power and the power carried by the scattered field alone and equation (13) still holds. However, one must now note that, in order to calculate the power absorbed by the scatterer as is done in equation (10), the source of the incident field must lie entirely outside the surface Σ of integration. Since any incident field containing evanescent components must be generated at finite distances from the scatterer, the surface Σ cannot simply be taken as large as we like as is done in calculations involving asymptotic methods for incident fields which are homogeneous. The source of homogeneous fields may be regarded as being located infinitely far from the scatterer and thus always outside Σ. Thus Σ must lie at a finite distance from the scatterer and we abandon the asymptotic methods employed to derive the theorem for homogeneous fields.

We can determine the power extinguished from an incident evanescent wave by identifying the variables in which the extinguished power must be analytic and then proceed to continue analytically, in several variables, the result pertaining to incident homogeneous waves.

It has been shown [6] that, if the unit vector $\mathbf{s}_j$ is expressed in polar coordinates (θ_j, ϕ_j),

$$\mathbf{s}_j = \hat{\mathbf{x}} \sin\theta_j \cos\phi_j + \hat{\mathbf{y}} \sin\theta_j \sin\phi_j + \hat{\mathbf{z}} \cos\theta_j, \tag{16}$$

where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are the unit vectors along the x, y and z directions respectively, then the scattering amplitude $f(\mathbf{s}_1, \mathbf{s}_2)$ is an analytic function of θ_1. Furthermore, in view of the reciprocity relation which holds for both real and complex directions of propagation [7],

$$f(\mathbf{s}_1, \mathbf{s}_2) = f(-\mathbf{s}_2, -\mathbf{s}_1), \tag{17}$$

the scattering amplitude $f(\mathbf{s}_1, \mathbf{s}_2)$ must also be an analytic function of θ_2. The complex unit vectors corresponding to the evanescent components of the field may be expressed in terms of complex θ and real ϕ. The analytic continuation of $f(\mathbf{s}_1, \mathbf{s}_2)$ to complex directions of propagation is unique and unambiguous and gives us a complete descrption of the scattered field including the near zone.†

Let us consider the power extinguished from a coherent beam consisting of two plane waves

$$\psi^{(i)} = a_1 \exp(ik\mathbf{r}\cdot\mathbf{s}_1) + a_2 \exp(ik\mathbf{r}\cdot\mathbf{s}_2). \tag{18}$$

† We refer the reader to [6], specifically equation (2.10). The amplitude in that reference is related to the scattering amplitude in our notation by $A(\alpha, \beta) = f(\hat{\mathbf{z}}, \mathbf{s})$ where $\mathbf{s}$ is given by equation (2.9) of [6]. For applications of analytic continuation to the calculation of the scattered field and the extinction coefficient associated with the reaction of evanescent waves, see references [8].

We denote the extinguished power in this case as $P^{(e)}(a_1, a_2)$. Making use of equation (13)) this quantity may be expressed in the form

$$P^{(e)}(a_1, a_2) = -r\Re \int_{(4\pi)} \{(\mathbf{s}+\mathbf{s}_1^*)a_1a_1^* f(\mathbf{s}, \mathbf{s}_1)\exp[ikr\cdot(\mathbf{s}-\mathbf{s}_1^*)]$$
$$+ (\mathbf{s}+\mathbf{s}_2^*)a_2a_2^* f(\mathbf{s}, \mathbf{s}_2)\exp[ikr\cdot(\mathbf{s}-\mathbf{s}_2^*)]$$
$$+ (\mathbf{s}+\mathbf{s}_2^*)a_1a_2^* f(\mathbf{s}, \mathbf{s}_1)\exp[ikr\cdot(\mathbf{s}-\mathbf{s}_2^*)]$$
$$+ (\mathbf{s}+\mathbf{s}_1^*)a_2a_1^* f(\mathbf{s}, \mathbf{s}_2)\exp[ikr\cdot(\mathbf{s}-\mathbf{s}_1^*)]\}\cdot\mathbf{s}\,\mathrm{d}\Omega(\mathbf{s}), \tag{19}$$

where $\mathrm{d}\Omega(\mathbf{s}) = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$, θ and ϕ being the polar coordinates of the unit vector $\mathbf{s}$. Because the real part of the integral is taken, this quantity is not analytic in the angular variables θ_1 or θ_2. In order to exploit the analytic properties of the scattering amplitude, it will prove useful to define a quantity

$$\mathcal{P} = P^{(e)}(1, \mathrm{i}) - P^{(e)}(1, -\mathrm{i}) + \mathrm{i}[P^{(e)}(1, 1) - P^{(e)}(1, -1)]. \tag{20}$$

Using equation (19) which applies for real as well as complex $\mathbf{s}_1$ and $\mathbf{s}_2$, we find that

$$\mathcal{P} = -2\mathrm{i}r \int_{(4\pi)} \{(\mathbf{s}+\mathbf{s}_1^*) f(\mathbf{s}, \mathbf{s}_2)\exp[ikr\cdot(\mathbf{s}-\mathbf{s}_1^*)]$$
$$+ (\mathbf{s}+\mathbf{s}_2) f^*(\mathbf{s}, \mathbf{s}_1)\exp[-ikr\cdot(\mathbf{s}-\mathbf{s}_2)]\}\cdot\mathbf{s}\,\mathrm{d}\Omega(\mathbf{s}), \tag{21}$$

for all, generally complex, unit vectors $\mathbf{s}_1$ and $\mathbf{s}_2$.

In view of the analytic properties of $f(\mathbf{s}_1, \mathbf{s}_2)$ it follows from equation (21) and Theorem 4.9.1 of [9] that $\mathcal{P}$ is an analytic function of the two complex variables θ_1^* and θ_2 separately. By Hartog's theorem [10, theorem 2, p. 32] it follows that $\mathcal{P}$ *is analytic in the space of the two complex variables* θ_1^* *and* θ_2.

The angular correlation function of the incident field consisting of two monochromatic plane waves defined by equation (18) is given by the expression

$$\mathcal{A}(\mathbf{s}', \mathbf{s}'') = [a_1\delta^{(2)}(\mathbf{s}'-\mathbf{s}_1) + a_2\delta^{(2)}(\mathbf{s}'-\mathbf{s}_2)]^*$$
$$\times [a_1\delta^{(2)}(\mathbf{s}''-\mathbf{s}_1) + a_2\delta^{(2)}(\mathbf{s}''-\mathbf{s}_2)], \tag{22}$$

where $\delta^{(2)}$ is the two-dimensional Dirac delta function. It follows from equation (1) that, for real $\mathbf{s}_1$ and $\mathbf{s}_2$,

$$P^{(e)}(a_1, a_2) = \frac{4\pi}{k}\Im[|a_1|^2 f(\mathbf{s}_1, \mathbf{s}_1) + a_1^* a_2 f(\mathbf{s}_1, \mathbf{s}_2) + a_2^* a_1 f(\mathbf{s}_2, \mathbf{s}_1) + |a_2|^2 f(\mathbf{s}_2, \mathbf{s}_2)]. \tag{23}$$

Using equation (23) and the definition (20) we find that

$$\mathcal{P} = \frac{8\pi}{k}[f(\mathbf{s}_1, \mathbf{s}_2) - f^*(\mathbf{s}_2, \mathbf{s}_1)], \tag{24}$$

for real $\mathbf{s}_1$ and $\mathbf{s}_2$.

Since $\mathcal{P}$, given by equation (21), is an analytic function of θ_1^* and θ_2 the expression on the right-hand side of equation (24) is the boundary value on the real θ_1^* and θ_2 axes of an analytic function of two complex variables. We seek the continuation of equation (24) in the space of complex θ_1^* and θ_2.

We recall that if $g(z)$ is an analytic function of z, then so is $g^*(z^*)$ and that $g^*(z)$ is an analytic function of z^*. We see that there is precisely one function, analytic in

θ_1^* and θ_2, whose boundary value on the real θ_1^* and θ_2 axes is equation (24), namely the function

$$\mathcal{P} = \frac{8\pi}{k}[f(\mathbf{s}_1^*, \mathbf{s}_2) - f^*(\mathbf{s}_2^*, \mathbf{s}_1)]. \tag{25}$$

The uniqueness of this continuation follows from a basic theorem of analytic function theory.

If we take $\mathbf{s}_1 = \mathbf{s}_2 = \mathbf{s}_0$, we deduce from the definition (20) that

$$\mathcal{P} = 4\mathrm{i}P^{(\mathrm{e})}(1, 0). \tag{26}$$

The power extinguished from a single incident plane wave, be it evanescent or homogeneous, is therefore given by the expression

$$P^{(\mathrm{e})} = \frac{4\pi}{k}\Im f(\mathbf{s}_0^*, \mathbf{s}_0). \tag{27}$$

Making use of the linearity of the problem, as was done in [4] to obtain equation (1), we find that the generalized optical theorem with arbitrary incident fields has the form

$$P^{(\mathrm{e})} = \frac{4\pi}{k}\Im \int\int \mathcal{A}(\mathbf{s}'_\perp, \mathbf{s}''_\perp) f(\mathbf{s}'^*, \mathbf{s}'')\, \mathrm{d}^2s'\, \mathrm{d}^2s'', \tag{28}$$

which reduces, as it should, to equation (1) when the incident field does not contain evanescent waves.

3. Physical interpretation

When the incident field is homogeneous, equation (27) has a well-known physical interpretation. Conservation of energy requires that the extinguished power $P^{(\mathrm{e})}$ must be removed from the incident field, evidently by an interference mechanism. As one moves progressively farther from the scatterer, there is only one component of the scattered field which propagates along with the incident plane wave, namely the forward-scattered field. *We may therefore conclude without any calculation that $P^{(\mathrm{e})}$ must be a function of the forward-scattering amplitude $f(\mathbf{s}_0, \mathbf{s}_0)$.* The exact functional relationship has to be worked out and is given by equation (27) with $\mathbf{s}_0$ being real for homogeneous waves [1–4].

If the incident wave is evanescent, the physical picture becomes more complicated. Evanescent waves cannot exist in a source-free unbounded three-dimensional space. We consider then, as a particular model for the generation of evanescent waves, two half-spaces, the left half-space being uniformly filled with a dielectric with a real index of refraction greater than unity at the frequency which we consider. The right half-space is taken to be vacuum, except for the presence of a scatterer of finite extent. In the absence of a scatterer, a single evanescent wave may be produced in the right half-space by the total internal reflection of a homogeneous plane wave incident from the left. The interaction of the evanescent wave and the scatterer produces a scattered field that carries a finite (non-zero) amount of power to the far zone.† Furthermore, the scatterer itself may absorb some of the incident power.

† The scattering amplitude $f(\mathbf{s}_1, \mathbf{s}_2)$ in this case must take into account the presence of the dielectric half-space. Within the accuracy of the first Born approximation (single scattering) the scattering amplitude is unchanged from that of the scatterer in free space.

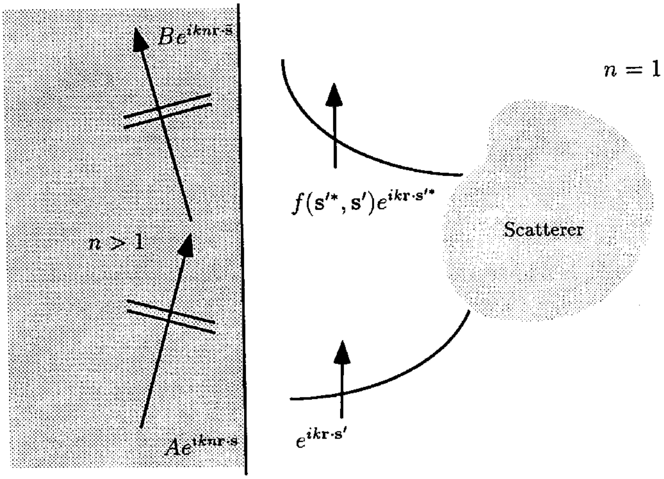

Figure 1. An evanescent plane wave in the right half-space is generated by total internal reflection of a wave which propagates in the direction specified by the unit vector $\mathbf{s} = p\hat{\mathbf{x}} + q\hat{\mathbf{y}} + m\hat{\mathbf{z}}$ in the left half-space. The resulting evanescent wave corresponds to a complex direction of propagation specified by the unit vector $\mathbf{s}' = np\hat{\mathbf{x}} + nq\hat{\mathbf{y}} + \mathrm{i}m'\hat{\mathbf{z}}$ with $m' = (n^2p^2 + n^2q^2 - 1)^{1/2}$, assuming that $n^{-2} < p^2 + q^2$. The reflected wave propagates in the direction specified by the unit vector $\tilde{\mathbf{s}} = p\hat{\mathbf{x}} + q\hat{\mathbf{y}} - m\hat{\mathbf{z}}$. A and B are constants which depend on the value of the index of refraction in the left half-space. The process of scattering generates another evanescent wave which corresponds to a complex direction of propagation $\mathbf{s}'^*$ and couples back into the dielectric half-space as a homogeneous plane wave which propagates in the direction of the reflected wave, $\tilde{\mathbf{s}}$.

We must account for the power scattered and absorbed by the scatterer in the right half-space by depletion of the *reflected* beam in the left half-space. *There is precisely one component of the scattered field which may couple back into the left half-space and propagate concurrently with the reflected beam and that is the evanescent component of the scattered field corresponding to the complex direction of propagation specified by* $\mathbf{s}_0^*$. The power must therefore be a function of the scattering amplitude in the direction specified by $\mathbf{s}_0^*$ for an incident evanescent plane wave with complex direction specified by $\mathbf{s}_0$ (figure 1). Equation (27) expresses this relationship.

There are other ways that one might envisage the generation of evanescent plane waves. One might consider the evanescent wave to be a component of a field produced by a source near the scatterer which does not itself appreciably interact with the scattered field. It has been shown that an evanescent wave may also be represented as a singular limit of a superposition of homogeneous plane waves, a particular example being the situation where evanescent waves are generated in the waist plane of a narrow Gaussian beam [11].

4. Examples of applications

To illustrate the main result derived in section 2, we consider the scattering of an evanescent wave by a dielectric medium characterized by a dielectric susceptibility $\eta(\mathbf{r})$ so that

$$\nabla^2\psi(\mathbf{r}) + k^2\psi(\mathbf{r}) = -4\pi k^2\eta(\mathbf{r})\psi(\mathbf{r}). \tag{29}$$

Assuming that the susceptibility is small compared with unity and from point to point changes smoothly throughout the scatterer, we may calculate the power perturbatively (i.e. by means of the Born series). We find that, to first order in η, the power extinguished from a single plane wave is given by the expression

$$P^{(e)} = 4\pi k \Im \int \eta(\mathbf{r}') \exp[-ik\mathbf{r}' \cdot (\mathbf{s}_0^* - \mathbf{s}_0)] \, d^3r'. \tag{30}$$

When the incident field is a homogeneous plane wave, the extinguished power yields information only about the volume integral of the imaginary part of the susceptibility. We can see that, when the incident field is evanescent, the extinguishing power is related to a component of the analytic continuation of the Fourier transform of the imaginary part of the suseptibility. If it is known *a priori* that the medium may be described by a function which is separable in the spatial variables in the form

$$\Im\eta(\mathbf{r}) = \alpha(z)\beta(\boldsymbol{\rho}), \tag{31}$$

where z is the distance along the direction of decay of the evanescent wave and $\boldsymbol{\rho}$ is a vector orthogonal to the z axis, then, by performing experiments with evanescent waves which decay at various rates, we may construct a Laplace transform of the z dependence of the imaginary part of the susceptibiility, that is the z dependence of the absorptive part $\alpha(z)$ of the medium. As a specific example of this situation, consider a slab of material whose susceptibility varies smoothly from zero to some maximum value and then back to zero over the thickness of the slab (this is in keeping with the conditions required for the validity of the first Born approximation), for example

$$\alpha(z) = \alpha_0 \sin\left(\frac{\pi z}{t}\right) \tag{32}$$

for $0 \leqslant z \leqslant t$ where t is the thickness of the slab. Writing

$$\int \beta(\boldsymbol{\rho}) \, d^2\rho = A\bar{\beta}, \tag{33}$$

we find that the power is given by the expression

$$P^{(e)} = 4ktA\bar{\beta}\alpha_0 \frac{1 + \exp(-k|s_0|t)}{1 + (k|s_{0z}|t)^2}, \tag{34}$$

which is proportional to the Laplace transform of $\alpha(z)$ with the variable of transformation being $k|s_{0z}|$ (figure 2).

We might also apply this result to scattering on a highly disordered non-absorbable scatterer. Explicitly, we consider the situation where

$$\Im\langle\eta(\mathbf{r})\rangle = 0 \tag{35}$$

and

$$C(\mathbf{r}, \mathbf{r}') = \Gamma_\eta(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}'), \tag{36}$$

where $C(\mathbf{r}, \mathbf{r}') = \langle\eta(\mathbf{r})\eta(\mathbf{r}')\rangle$ is the two-point correlation function of the susceptibility η.

It has been shown [4] that for such a medium the scattering amplitude to the second order of perturbation is given by the expression

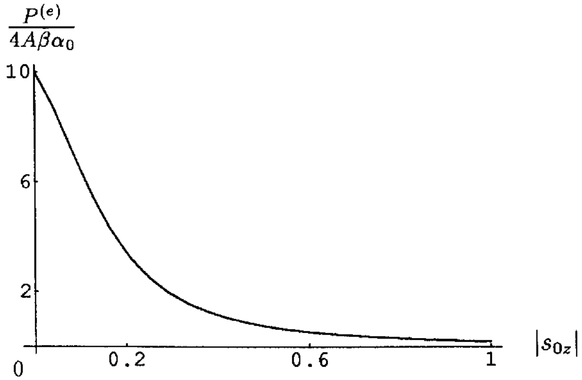

Figure 2. The extinguished power as a function of the decay constant of the evanescent wave, $|s_{0z}|$, for the scatterer described by equation (32) with the wavelength of the incident field being such that $kt = 5$.

$$f(\mathbf{s}, \mathbf{s}_0) = k^4 \int_V \int_V C(\mathbf{r}', \mathbf{r}'') \frac{\exp(\mathrm{i}k|\mathbf{r}' - \mathbf{r}''|)}{|\mathbf{r}' - \mathbf{r}''|} \times \exp[-\mathrm{i}k(\mathbf{s} \cdot \mathbf{r}' - \mathbf{s}_0 \cdot \mathbf{r}'')] \, \mathrm{d}^3 r' \, \mathrm{d}^2 r''. \tag{37}$$

Using equation (37) and (27) we see that in this case

$$P^{(e)} = 4\pi k^4 \int_V \Gamma_\eta(\mathbf{r}') \exp[-\mathrm{i}k(\mathbf{s}_0^* - \mathbf{s}_0) \cdot \mathbf{r}'] \, \mathrm{d}^3 r', \tag{38}$$

which gives the familiar result that, when the incident field is homogeneous, the extinguished power is proportional to the zero spatial frequency component of the fluctuations of the medium. When the incident field is evanescent, the extinguished power is related to a component of the analytic continuation of the Fourier transform of the spatial fluctuations of the medium.

In both examples given here, the information obtained in measuring the extinguished power may be useful in calculating a super-resolved image of the scattering object. Traditional methods of super-resolution rely, either explicitly or implicitly, on analytic continuation of the Fourier components of the scatterer outside the Ewald limiting sphere. These methods are computationally unstable because of the exponential growth of noise in the process. The ability to measure the Fourier transform of the object function at points along the imaginary axes of the complex Fourier variables may allow for a check against run-away exponential errors.

5. Conclusion

We have found a generalization of the optical theorem to situations in which the incident wave is evanescent. In most practical situations the evanescent components of the incident field may be neglected because they fall off exponentially with increasing propagation distance. However, there are some situations, notably in near-field optics, in which the scatterer is in close proximity to a source of inhomogeneous fields. In these situations the evanescent components of the incident field may interact with the scatterer and contribute significantly to the power radiated to the far field.

Acknowledgments

The author would like to thank Dr Emil Wolf for careful reading of several drafts of this paper and for many helpful discussions and comments. He is also obliged to Dr Boris Zel'dovich for some helpful comments and discussions. This research was supported by the US Department of Energy under grant DE-FG02-90ER 14119 and by the Air Force Office of Scientific Research under grant F49620-96-1-0400.

References

[1] Feenberg, E., 1932, *Phys. Rev.*, **40,** 40.
[2] van der Hulst, H. C., 1949, *Physica*, **15,** 740.
[3] Newton, R. G., 1982, *Scattering Theory of Waves and Particles* (New York: Springer).
[4] Carney, P. S., Wolf, E., and Agawal, G. S., 1997, *J. opt. Soc. Am.* A, **14,** 3366.
[5] Bleistein, N., and Handelsman, R. A., 1975, *Asymptotic Expansion of Integrals* (New York: Holt, Rinehart and Winston).
[6] Wolf, E., and Nieto-Vesperinas, M., 1985, *J. opt. Soc. Am.* A, **2,** 886.
[7] Carminati, R., Nieto-Vesperinas, M., and Griffet, J., 1998, *J. opt. Soc. Am.* A, **15,** 706.
[8] Quinten, M., Pack, A., and Wannemacher, R., 1999, *Appl. Phys.* B, **68,** 87; Zvyagin, A. V. and Goto, K., 1998, *J. opt. Soc. Am.* A, **15,** 3003; Liu, C., Kaiser, J., Lange, S. and Van Labeke, D., 1993, *J. mod. Opt.*, **40,** 1239; Chew, H., Wang, D.-S. and Kerker, M., 1979, *Appl. Opt.*, **18,** 2679.
[9] Dettman, J. W., 1965, *Applied Complex Variables* (New York: Dover Publications).
[10] Bochner, S., and Martin, W. T., 1948, *Several Complex Variables* (Princeton Univrsity Press).
[11] Berry, M., 1994, *J. Phys.* A, **27,** L391.